Epidemics in Networks Part I — Introduction

Joel C. Miller & Tom Hladish

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1/52

Introduction

Disease spread

Key Questions

Modeling approaches

Networks

Brief glance at SIR in networks

Random network models

Real world networks

Review

References

Who are we?

- ► Joel C. Miller:
 - Former math and biology faculty at Penn State and later Monash University (Melbourne).
 - Now senior research scientist at Institute for Disease Modeling
 - Co-author of "Mathematics of Epidemics on Networks": http://bit.ly/EpidemicSonnetWorks
 - Developer of python package EoN: http: //epidemicsonnetworks.readthedocs.io/en/latest/
 - 8th year teaching this course.
- Thomas J. Hladish
 - Biology and Emerging Pathogens Institute faculty at the University of Florida
 - Developer of C++ EpiFire, AbcSmc packages: https://github.com/tjhladish/
 - 10th year teaching this course

Layout of course

The course will consist of a mixture of theory and computer labs.

- Theory
 - Properties of diseases and networks
 - Analytic predictions of disease behavior
- Computer Lab
 - Python and EpiFire-based stochastic simulation of epidemics on networks.
- Notes are available at http://sismid.hladish.com

Introduction

Disease spread

- Key Questions
- Modeling approaches
- Networks
- Brief glance at SIR in networks
- Random network models
- Real world networks
- Review
- References

Two features primarily determine population-scale disease spread:

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Population structure.

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Population structure.

Immune response / natural history.

Immune response determines result of individual's exposure and whether onwards transmission occurs.

Possible outcomes:

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Possible outcomes:

Remains infected forever: SI



HIV, Tuberculosis (without treatment), Hepatitis (sometimes),

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- Remains infected forever: SI
- Gains permanent immunity: SIR



Measles, Mumps, Rubella, Pertussis, ...

Immune response determines result of individual's exposure and whether onwards transmission occurs.

Possible outcomes:

- Remains infected forever: SI
- Gains permanent immunity: SIR
- Recovers but can be reinfected: SIS



Many parasites (e.g., lice), Many bacteria, Many STDs, ...

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Possible outcomes:

- Remains infected forever: SI
- Gains permanent immunity: SIR
- Recovers but can be reinfected: SIS
- Recovers with temporary immunity: SIRS



Dengue (sort of), Pertussis, Influenza (because of genetic drift of virus).

Immune response determines result of individual's exposure and whether onwards transmission occurs.

Possible outcomes:

- Remains infected forever: SI
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- Recovers with temporary immunity: SIRS

Introduction

Disease spread

Key Questions

Modeling approaches

Networks

Brief glance at SIR in networks

Random network models

Real world networks

Review

References

Lots of things to think about

For SIR, we are typically interested in

- \mathcal{P} , the probability of an epidemic.
- ► A, the "attack rate": the fraction infected (better named the attack ratio)
- ▶ R₀, the average number of infections caused by those infected early in the epidemic.
- I(t), the time course of the epidemic.

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For SIS, we are typically interested in

- $\blacktriangleright \mathcal{P}$
- $I(\infty)$, the equilibrium level of infection
- ► R₀
- ► I(t)

Introduction

Disease spread

Key Questions

Modeling approaches

Networks

Brief glance at SIR in networks

Random network models

Real world networks

Review

References



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Simple Compartmental Models

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We will analyze compartmental models in detail later to provide context for network models.

Introduction

Disease spread

Key Questions

Modeling approaches

Networks

Brief glance at SIR in networks

Random network models

Real world networks

Review

References

A <u>network</u> is a collection of individuals joined together based on interactions that may spread the disease in question. These connections (edges) may be:

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Other important types of networks

The nodes don't have to be individuals. They can be communities.

- Airline network
- Connected communities
- Livestock movement

▶ ...

Network definition

- A <u>network</u> is a collection of <u>nodes</u> which are joined into pairs by <u>edges</u>.
- Two nodes that are joined together are called <u>neighbors</u>. The number of neighbors a given node has is its degree, k.
- There is no real difference between the definitions of "network" and "graph".
- I will tend to use the terminology "partner" for neighbor and "partnership" for "edge" [the term "contact" is also commonly used but can be ambiguous].
Network Properties

There are a number of things we can measure:

▶ Degree distribution: *P*(*k*), the proportion of nodes with degree *k*.

High degree nodes tend to be infected early and in turn infect more nodes. So the early growth is more affected by the presence of high-degree nodes than by the average degree.

Network Properties

There are a number of things we can measure:

- Degree distribution: P(k), the proportion of nodes with degree k.
- Clustering: frequency of short cycles [not common in sexual networks].

Clustering tends to slow the spread of a disease, but often does not significantly affect whether a disease occurs or how large it gets. Its role is reduced as typical degrees increase.

Network Properties

There are a number of things we can measure:

- Degree distribution: P(k), the proportion of nodes with degree k.
- Clustering: frequency of short cycles [not common in sexual networks].
- Partnership duration: Network may be dynamics, with partnerships changing in time. Individuals may enter/leave the population.

Changing partnerships reduces the effect of local "susceptible depletion"

More Network Properties

There are a number of things we can measure:

Edge weights: some edges may have higher transmission probabilities than others.

Edge weights and many other effects are generally less significant (but what if weights inversely correlated with degree?)

More Network Properties

There are a number of things we can measure:

- Edge weights: some edges may have higher transmission probabilities than others.
- Assortativity: Individuals may actively select similar partners. In particular, partners with similar degree.

Assortative mixing by degree tends to make it easier for a disease to get established because the core of high-degree nodes provides a good place to spread. However, it often reduces the total size of the epidemic because the low degree nodes tend to connect only to low degree nodes.

More Network Properties

There are a number of things we can measure:

- Edge weights: some edges may have higher transmission probabilities than others.
- Assortativity: Individuals may actively select similar partners. In particular, partners with similar degree.
- Modularity: some parts of the network may be more densely connected than others.

The existence of subcommunities may influence how a disease spreads (and when it is detected).

Introduction

Disease spread

Key Questions

Modeling approaches

Networks

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Review

References

A preliminary glance at SIR disease in networks

To give context to our first computer simulations, we take a preliminary look at SIR disease in a simple network.

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A preliminary glance at SIR disease in networks

- To give context to our first computer simulations, we take a preliminary look at SIR disease in a simple network.
- We take a very simple network: 4 nodes in a line.
- We take a very simple SIR disease: in a time step nodes transmit to neighbors with probability p and then recover with immunity.



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21 / 52















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Introduction

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Key Questions

Modeling approaches

Networks

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Review

References

Random networks

We rarely have exact data about a population's contact structure. Instead we have measurements of a few important features.

- We want to generate a random network that captures these properties.
- If these properties are the relevant properties, then disease spread in the simulated network will accurately reproduce dynamics in the real population.
- Ideally we can analytically predict the dynamics in the model network.
- An interesting challenge [outside our scope] is finding ways to generate random networks with specified properties which are analytically tractible.

Common random network models

- Small World networks
- Barabási-Albert
- Configuration Model
- Exponential Random Graph Model [ERGM]

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- Barabási-Albert
- ► Configuration Model ← Analytically Tractible
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Small-world networks







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Small-world networks



- Start with nodes in a ring and connect nearby pairs.
- Rewire a fraction p of the edges.
- The resulting network has short typical path lengths and high "clustering".

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Epidemics in Small-world networks: theory vs simulation



Epidemics in Small-world networks: theory vs simulation



27 / 52

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Barabási-Albert networks

- Start with m + 1 nodes all connected to each other.
- Add a node, connect it to m previously existing nodes
- Repeat, each time selecting the previously existing nodes with probability proportional to their degree.



Epidemics in BA networks: theory vs simulation



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Epidemics in BA networks: theory vs simulation



Configuration Model

Probably the simplest model capturing a heterogeneous degree distribution:



Configuration Model

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What can go wrong?

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What can go wrong? Create the graph at the last step: L = [7, 6, 5, 6, 2, 2, 4, 7, 1, 3, 6, 5, 4, 5]

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Configuration model networks, all degrees equal 4.

Density of "short" cycles is small for large N



Configuration model networks, all degrees equal 4.

"Annealed" Configuration Model



- The <u>annealed network</u> version assumes that at every moment the network looks like a Configuration model network.
- However, at every moment, an individual changes all of its partners.
- In practice this is appropriate if partnerships are so short or disease transmission so rare that an individual is unlikely to ever transmit to the same individual twice or transmit back to its infector.
- People who use the term "annealed network" call the static version a "quenched network".

Exponential Random Graph Model (ERGM)

 Given some vector of parameters θ and statistical measurements s on a graph G, choose G with probability proportional to

$\exp[\theta \cdot s]$

- Generally a network is chosen through MCMC.
- Computational power significantly constrains the network size

Do your friends have more friends than you do (on average)?

Given a configuration model network G with a heterogeneous degree distribution:

Do your friends have more friends than you do (on average)?

Given a configuration model network G with a heterogeneous degree distribution:

If we choose a random individual in a configuration model network, is its expected degree

- 1. higher
- 2. lower
- 3. the same
- 4. depends on the degree distribution

than the expected degree of a random partner?

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The probability a partner has degree k is $P_n(k) = kP(k)/\langle K \rangle$.

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- What about a random partner? What is P_n(k), the probability a partner has degree k?

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- A partner's partner also has degree k with probability $P_n(k)$.

I cannot stress enough that if P(k) is the probability a random individual has k partners, then

 $P_n(k) = kP(k)/\langle K \rangle$

is the probability a random partner has k partners.

Introduction

Disease spread

Key Questions

Modeling approaches

Networks

Brief glance at SIR in networks

Random network models

Real world networks

Review

References

Social networks

- facebook
- linkedin
- twitter
- ▶ ...

These may be more appropriate for spread of ideas or opinions.

Contact networks

- The network of physical interactions.
- Often highly clustered.
- Appropriate for respiratory diseases.
- Sometimes measured by giving people devices that measure physical proximity.

Sexual networks

- Appropriate for sexually transmitted diseases.
- Often low clustering.
- Often highly heterogeneous.
- Transient partnerships may play a large role.

Location-Location networks

- Cities connected by travel of people between them [spread of H1N1, Ebola].
- Farms connected by movement of animals [foot and mouth].
- Habitats connected by bird migrations [West Nile].
A number of attempts have been made to measure networks in "the wild". Each case has its own peculiarities. This list is a little dated.

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- Romantic networks [10]

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- Seasonal population movements [16]: study of seasonal population movements for malaria control (phone data, census, satellite imagery).

A number of groups have done large-scale simulations of populations

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- South Africa: Simulation by George Seage's group at HSPH for HIV transmission (≈ 6 million?)

Introduction

Disease spread

Key Questions

Modeling approaches

Networks

Brief glance at SIR in networks

Random network models

Real world networks

Review

References

Review

- Disease dynamics depend on immune response and population structure
- Simple SIR disease can be modeled through percolation.
- We will focus on "Configuration model" populations and hope that they are close enough to real populations.
- There is a size-biasing effect by which higher degree nodes are more likely to be infected early on and then transmit to more nodes.

Introduction

Disease spread

Key Questions

Modeling approaches

Networks

Brief glance at SIR in networks

Random network models

Real world networks

Review

References

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